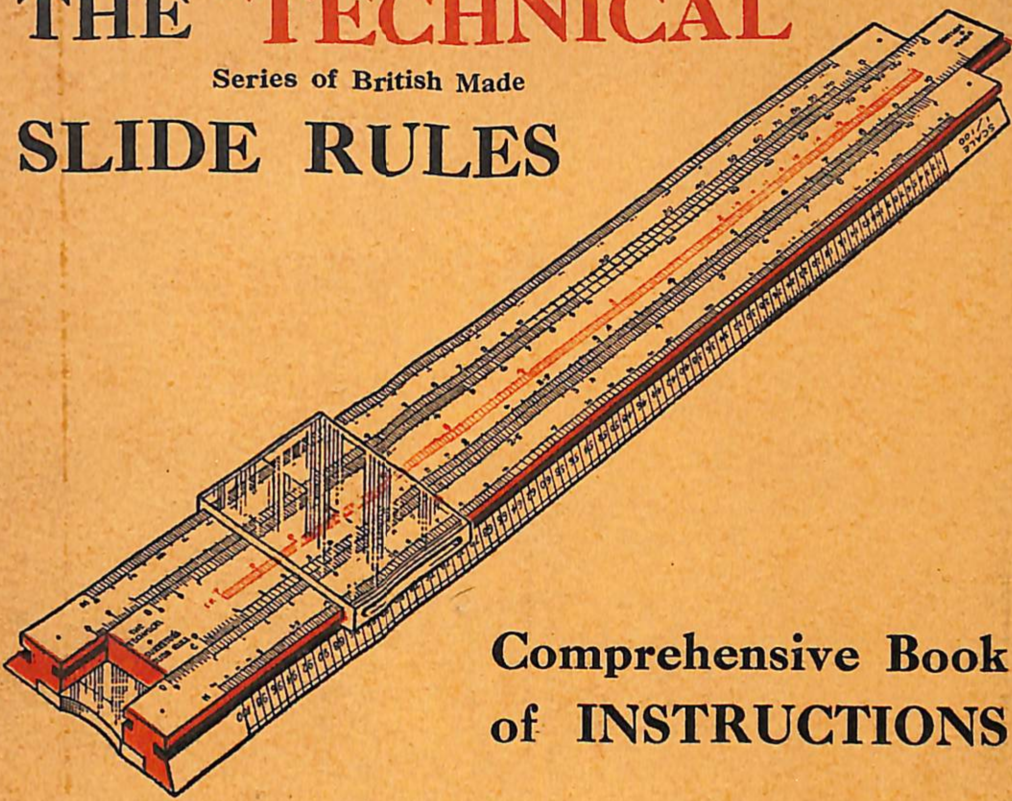
THE "TECHNICAL"

Series of British Made

SLIDE RULES



Comprehensive Book of INSTRUCTIONS

Contents :

THE STANDARD RULE for General Use

Elementary principles of the Slide Rule: Logarithms, Involution, Evolution, etc. Application of Engineers' Constants.

THE SURVEY RULE for Land Survey, Aerial and Sea Navigation

Solution of Plane and Spherical Triangles, Conversion Tables, Gunnery Formulae, Method of obtaining Ground Speed, Departure, Track, Distance, Rhumb Line, etc.

THE AERONAUTICAL RULE for Aeronautics and Electrical Use

Cubes and Cube Roots, Conversion Factors for KW and H.P. Efficiencies, Properties and Strengths of Material, Euler's Formulae, Aerodynamics.

TAKE CARE OF YOUR SLIDE RULE

1. **Always keep the Rule in its Case when not in use.**

2. **Never Grease the Slide or Stock, or allow it to get damp.**

3. **Do not expose to a Strong Sun or Excess Heat.**

If after constant use the Slide tends to run loose—bind it round tightly with a piece of tape or rubber band, and leave overnight. This will be found to tighten the springs and close the stock.

If the slide is too tight, ease the springs by bending the stock gently outwards, or sharply run the slide up and down the rule.

By carefully following these instructions your Slide Rule will last indefinitely.

PRELIMINARY NOTES

Users of the " TECHNICAL " Slide Rule should thoroughly acquaint themselves with the preliminary instructions and elementary principles before proceeding with the detailed instructions for any particular rule.

It will be found that some of the examples given in the sections dealing with the " Aeronautical " and " Surveyor's " Slide Rules may be worked on the " Standard " Slide Rule.

The " Technical " series of Slide Rules have been designed to cover a very wide range of calculations. The special feature of the " Surveyor's " Slide Rule is its handiness in dealing with problems involving Trigonometrical Ratios such as the calculations which have to be made every day in Navigation, both on the sea and in the air.

The " Aeronautical " Slide Rule can also be used for most calculations of the Navigational type and in addition, its special features render it of great value to the electrical and to the aeronautical engineer.

THE " TECHNICAL " STANDARD SLIDE RULE

The Slide Rule is an instrument principally used for the multiplication and division of numbers, and for those calculations that commonly occur in general and electrical engineering practice, survey and navigational work.

The operation is performed mechanically by the application of logarithmic addition and subtraction. The numbers are set on logarithmic scales, but the logarithms themselves do not appear in the process. Thus in the same way the application of logarithms simplifies arithmetical calculations, so the slide rule simplifies the method of applying logarithms by dispensing with the necessity of referring to logarithm tables.

The Slide Rule consists of three main parts :—

- (*a*) The stock or Body.
- (*b*) The reversible centre slide.
- (*c*) The cursor—the transparent sliding frame.

THE SCALES

There are four main scales on the rule, called respectively A, B, C, D. The upper scale A of the body is exactly similar to the upper scale B of the slide, while the lower scale C on the slide is repeated in the lower scale D on the body. The A and B scales consist of two exactly similar scales, which may be called the left-hand A (or B) scale, and the right-hand A (or B) scale respectively. This will be subsequently referred to as the L.H. for the left-hand or the R.H. for the right-hand Index respectively.

The C and D scales are similar to each of these scales, *i.e.*, the C and D scales are the same as each half of the A and B scales. It follows, therefore, that the C and D scales, being twice as long as the A and B scales, can be subdivided and read to a greater degree of accuracy. Note that the A and B scales read from 1—100, while the C and D scales read from 1—10.

Each of the scales is divided into 10 parts numbered 1—10, and these are again subdivided into 10 parts. Further subdivision is made according to the space available. On the C and D scales the interval between 1 and 2 is divided into 10 parts, and these are read as 11, 12, 13, etc., although they are marked 1, 2, 3, etc.

The 1 at the left may be taken as, 1, 10, 0·1, etc., the position of the decimal point only needing to be fixed for each particular calculation, the exact position of the decimal point in the final result is best obtained by using characteristics as in logarithmic calculations. This method is explained in detail in the " position of the decimal point."

In addition to the A, B, C and D scales the " Technical " Standard Slide Rule is equipped with the M and N log-log scales, and the Sine, Tangent and ST scales (the latter is used for obtaining the values of sines and tangents between 3′ and 5°43′). The use of these scales is explained in detail under their respective headings.

There are two linear scales extending from the sides of the rule. The scale marked from 1/10 represents a 10″ rule. The inch divisions are subdivided by thirty two, thus each division represents 1/32nd of an inch. The other scale represents 27 centimetres subdivided into millimetres. The use of these scales is apparent.

MULTIPLICATION

Since the C and D scales are twice the length of the A and B scales, and permit easier reading, they are more conveniently used for fractional calculations.

The following rules for multiplication are appliable when either the A B or C D scales are used, excepting that the result of C is read *below* on D, and the result of B is read *above* on A.

Rule.—Under one of the factors on the A scale set unity on the B scale, and over the other factor on the B scale read the product on the A scale.

Example 1 To multiply 20×2, set the 1 on the B scale under 20 on A scale—the product 40 is read on the A scale above 2 on the B scale. Note that 3, 4 and 5 on the B scale are respectively coincident with 60, 80 and 100 on the A scale.

Example 2 To multiply $3·0 \times 2·5$, set 1 on the C scale over 3 on the D scale, the product 7·5 is read on the D scale under ·25 on C. Note 3·5 on C is coincident with the extension of the D scale to be read as 10·5.

If the product is more than 10 the second number on the slide will be found to be beyond the rule. The right-hand end of the slide or the 10 in the C scale is used.

E.g., to multiply 3×6 set the 10 on the C scale above the 3 on the D scale and read off below the 6 on the C scale the result 18 on the D scale.

DIVISION

Rule.—Place the divisor on B scale under the dividend on the A scale, the quotient is read on the A scale above unity on B scale. In using CD scales the result is read *below* on D.

Example 1.—To divide 40 by 2 set 2 on the B scale under 40 on the A scale the quotient 20 is read on A scale above unity on B.

Note the result of $60 \div 3$ and $80 \div 4$ is obtained on the same setting of the scales.

Example 2.—To divide 7·5 by 2·5, set 2·5 on C scale over 7·5 on D scale, the quotient 3 is read on D scale under unity on C scale.

Note the setting is coincident for the quotient of $6 \div 2$; $9 \div 3$; $4·5 \div 1·5$.

To find the position of the decimal point in the result, the characteristics are subtracted as in ordinary logarithms. In this case when the 10 is used instead of the 1, then 1 is subtracted from the final characteristic.

THE POSITION OF THE DECIMAL POINT

This is best found from the characteristics as in logarithms. The characteristic of the product is equal to the sum of the characteristics of the factors. The only important point here is that, if instead of the 1 on the C scale the 10 is used, then 1 must be added to the characteristic sum.

E.g., when multiplying 3×6 the sum of the characteristics is $0 + 0 + 1$ since the 10 on the C scale is used.

When multiplying 0·039 by 4350 the sum of the characteristics is $- 2 + 3 + 1$ the 1 being added since the 10 on the C scale is used.

The position of the decimal point is usually obvious from the nature of the calculation.

COMBINED MULTIPLICATION AND DIVISION

Where an expression of the form $\dfrac{a \times b \times c \times d}{e \times f \times g}$ is to be evaluated, this may be done in one series of operations without actually evaluating either the numerator or denominator.

E.g., $$\frac{723 \times 2{\cdot}46 \times 36{\cdot}9 \times 314}{37 \times 4{\cdot}5 \times 32}$$

Put the 37 on the C scale above the 723 on the D scale and move the cursor to 246 on the C scale. Now bring the 45 on the C scale under the cursor line and move the cursor to 369 on the C scale. Bring the 32 on the C scale under the cursor line and move the cursor to 314 on the C scale. Now read off the result below the cursor line on the D scale, *i.e.*, 3865. Now to find the position of the decimal point. The sum of the characteristics in the numerator is $2 + 0 + 1 + 2 = 5$ and in the denominator $1 + 0 + 1 = 2$. The characteristic of the result is $5 - 2$ or 3. Therefore the result is 3865. Care must be taken to make allowance for the addition or subtraction of 1 to the characteristic when it is necessary to use the 10 of the C scale instead of the 1.

RATIO AND PROPORTION

If scales such as A and B or C and D be set in any relative position the ratio of cursor readings is constant to the setting.

Example 1.—Set 3 on the B scale below 2 on the A scale, then any set of cursor readings will be in the ratio 2 : 3. Thus 3–4·5 ; 4–6 ; 5–7·5 ; 6-9 ; 7–10·5 ; 9–13·5 ; 10–15.

Example 2.—Set 2·2 on the C scale above 4 on the D scale, the cursor readings will then be in the ratio 2·2 : 4, or 22 : 40. Thus 1·1–2 ; 11–20 ; 3·3–6 ; 33–60 ; 4·4–8 ; 44–80 ; 5·5–10 ; 55–100.

INVOLUTION AND EVOLUTION.

The numbers on the A and B scales are the squares of the numbers on the C and D scales, and these may be read off directly with the aid of the cursor.

E.g. The square of 3 on the D scale is read directly above on the A scale=9.

Similarly to find the square root of any number, the number is selected on the A scale and read off directly below on the D scale.

The characteristic of the square is double that of the number if the square is found on the top L.H. scale, and if it is on the R.H. top scale 1 must be added to twice the characteristic.

E.g., $18^2 = 324$ characteristic $2 \times 1 = 2$ L.H.
$52^2 = 2704$,, $2 \times 1 + 1 = 3$ R.H.

In finding the characteristic of square roots the following rule is to be applied. If the characteristic of the number is even, the top left-hand is used and the characteristic of the root is half that of the number. If the characteristic of the number is odd the right-hand top scale is used, and the characteristic of the root is $\frac{a-1}{2}$, where a is the characteristic of the original number.

The cubes of numbers can be obtained by placing 1 on C scale over the number on D scale, and the result is read on A scale above the number on B scale.

E.g. To find the cube of 4, place 1 on C over 4 on D, and read the result 64 on A scale over 4 on B scale.

Cube roots of numbers are obtained by setting the cursor on A scale over the number required. Now move the slide until the L.H. 1 of B scale is coincident to the same number that appears under the cursor line on C scale.

E.g. Find the cube root of 27.

Set cursor to 27 on A scale, then move slide until the left-hand 1 on B scale appears under the same number as that under the cursor. In this case it will be found that 3 is coincident, ∴ the answer=3. However, it will be found that cube and other roots are best evaluated with the aid of the log-log scales.

Users of the " Technical " Aeronautical Rule can obtain the cube and cube root of any number directly with the aid of the cursor.

RECIPROCALS

By reversing the slide in the groove so that the scales now run ACBD, the reciprocals of numbers, *i.e.*, the value of 1 divided by the number may be obtained. To find the reciprocal of any number place the cursor line over the number on the C scale and read off the result under the line on the D scale,

$$E.g.,\ \frac{1}{2{\cdot}7}=0{\cdot}37.$$

First reverse the slide and arrange the 10 on the C scale in line with the 1 on the A scale. Now place the cursor line on 2·7 on the C scale and read off under the line on the D scale the result 0·37.

The " TECHNICAL " Aeronautical and Surveyor's Rules are equipped with a C scale in reverse, running down the centre of the slide. This scale is referred to as the CR scale. The full advantage of this scale is explained in detail under its respective heading in the Survey section of this book.

LOG-LOG SCALES

These are the two scales on the top and bottom of the face of the rule, called the M and N scales. The M scale runs from 1·1 to 2·9, and the N scale from 2·6 to 50,000.

The 10th power of all numbers on the M scale can, with the aid of the cursor, be read directly on the N scale. Thus the 10th root of all numbers on the N scale can be obtained by reading directly on the M scale.

Using the cursor and the C scale, expressions of the form a^x or $\sqrt[x]{\frac{1}{a}}$ may be evaluated even when a or x are not whole numbers.

A special index mark W is used on the C scale such that the distance 1 to W is equal to the length from 1·1 to 2·9. This is only used for rules where the N scale reads from 2·9 to 100,000.

Example 1.—(*a*) Where the result is less than 2·9, *e.g.*, $1{\cdot}26^{2.4} = 1{\cdot}742$ set the cursor to 1·26 on the M scale and bring the 1 on the C scale under the cursor line. Then set the line on the 2·4 on the C scale and read off the result under the line on the M scale.

(*b*) Where the result is greater than 2·9, *e.g.*, $2{\cdot}36^{2.16} = 6{\cdot}39$, set the cursor to 2·36 on the M scale and bring the 10 on the C scale under the cursor line. Then set the line on 2·16 on the C scale and read off the result under the line on the N scale.

(*c*) Where the number whose power to be found is greater than 2·9, *e.g.*, $3{\cdot}9^{3.5} = 118$, this is similar to (*a*) except that the N scale is used.

Example 2.—(*a*) Where the root is greater than 2·9, *e.g.*, $\sqrt[3{\cdot}04]{47} = 3{\cdot}545$, set the cursor to 47 on the N scale and bring the 3·04 on the C scale under the cursor line. Then set the cursor line on the 1 of the C scale and read off the result under the line on the N scale.

(*b*) Where the root is less than 2·9, *e.g.*, $\sqrt[7{\cdot}6]{9{\cdot}2} = 1{\cdot}339$, set the cursor to 9·2 on the N scale and bring the 7·6 on the C scale under the cursor line. Then set the cursor line on the 10 of the C scale and read off the result under the line on the M scale.

COMMON LOGARITHMS

To assist in obtaining the common log of any number on the upper or lower log-log scales, the mark L is positioned on the C scale equivalent to 2·3026. The mark L is placed coincident to the number on the log-log scale and the result read off on D scale below 1 (or 10) of the C scale.

Example 1.—To find the common log of 21·25, place the cursor line over 21·25 in the lower log-log scale and coincide the mark L in C scale. The result 1·3273 is read opposite 1 in C on the D scale.

Example 2.—To find the common log of 1·353 place the cursor line over 1·353 in the upper log-log scale and coincide the mark L. The result ·1313 is read opposite 1 in C on the D scale.

THE GAUGE MARKS

In addition to the constant L, the following series of gauge points are marked on the face of the " TECHNICAL " Standard Slide Rule.

The mark π is positioned on the A, B, C and D scales at 3·1416 and is the well known constant for calculating the circumference of a circle given the diameter, or *vice versa*.

The mark M is positioned at 0·3183 on the R.H. of the A and B scales only, and gives the value of $\frac{1}{\pi}$, which is the reciprocal of π.

The marks A and A′ are also found on the upper scales, these denote the value of $\frac{\pi}{4}$, and both should therefore be read as 0·7854.

The marks C and C′ are positioned at 1·128 and 3·568 respectively on the C and D scales only. These represent the values of $\sqrt{\frac{4}{\pi}}$ and $\sqrt{\frac{40}{\pi}}$.

It will be found that when these marks are projected into the A and B scales the reading for both is 1·273, which equals the value of $\frac{4}{\pi}$ or the reciprocal of $\frac{\pi}{4}$.

The use of these points is apparent when the principle of substituting the reciprocal for the multiplier is used when evaluating an expression where π or $\frac{\pi}{4}$ appears in the numerator.

Example 1. The volume of a cylinder can be calculated from the formulae $\frac{\pi}{4}ld^2$, where $l =$ length and $d =$ diameter of the cylinder.

By substituting $\frac{\pi}{4}$ for its reciprocal $\frac{4}{\pi}$ the formulae now becomes

$$\frac{d^2}{1{\cdot}273} \times l, \text{ or, } \frac{ld^2}{\text{C}}$$

This can now be easily obtained in one setting of the slide. Set the gauge mark C on the C scale over the diameter (d) on the D scale, and over the length (l) on B scale read the volume on A scale.

Example 2. It is required to find the curved surface of a cylinder. The formulae used in this case is $l \times d \times \pi$. By substituting π for its reciprocal $\frac{1}{\pi}$ the formulae now becomes :—

$$\frac{ld}{\frac{1}{\pi}} \text{ or, } \frac{ld}{\text{M}}$$

This also can be evaluated in one setting of the slide. Coincide M on B to the diameter (d) on A scale, and over the length (l) on B scale read the answer on A scale.

Example 3. The formulae required to find the area of a circle equals

$$\text{d}^2 \times \frac{\pi}{4} = \text{d}^2 \times 0{\cdot}7854 = \text{d}^2 \times \text{A (or A}')$$

Thus to find the area of a circle given diameter = 4. Set R.H. 1 of C scale over the diameter (4) on D scale and above A′ on B scale read the answer 12·567 on A scale. Note here that if the L.H.1 of the C scale was used in the example the result could only have been obtained by crossing the slide. Therefore, the R.H.1 was employed and the result read above A′. The position of the decimal point in this case was obvious, and in other cases is usually apparent by the term of the diameter.

SINE and TANGENT SCALES.

These scales are placed on the reverse of the slide and the values of the Sines, Tangents and sines and tangents of small angles, can be read in conjunction with their respective scales by use of the L.H. or R.H. back index windows.

The value of the Sine is read in conjunction with the A and B scales. The left hand reads from 0·01 to 0·1 (sin 0°35′ to sin 5°43′), while the right hand reads from 0·1 to 1·0 (sin 5°43′ to sin 90°).

E.g., Sin 27° = 0·455.

Set 27 on the S scale to R.H. index window, and read the result on B scale as 455 under R.H.1 on A scale.

Using the left hand, *e.g.* Sin 4°30′ = 0·0787.

Set 4°30′ on S scale to L.H. index window, and read the result as 787 on B scale under L.H.1 on A scale. (Note that since the value of the sine appears in the left hand an 0 is placed immediately after the decimal point).

The value of the Tangent is read in conjunction with the C and D scales from 0·1 to 1·0 (tan 5°43′ to tan 45°).

E.g., tan 12° = 0·2124.

Set 12 on the T scale to L.H. index window, and read the result on C scale over L.H.1. on D scale.

The same method is used to obtain the values on the ST scale. This scale is graduated from 3′ to 5°43′. Since the values of the Sine and Tangent for small angles are nearly equal (the difference being in the 4th decimal place) natural sines or tangents between 3′ and 5°43′ can be read off on the A scale.

Considering only that part of the ST scale between 3′ and 35′ it will be seen that a third power of 10 has been introduced since sin 3′ = 0·001 ; sin 34′ = 0·01. This portion of the ST scale performs the same function as if the A and S scales had both been extended to the left. Thus the readings would be :—

A scale........................	·1........................	1 ————	10————		100
S scale————	sin 3′....................	sin 34′...............	sin 5° 43′................		sin 90°

It can, therefore, be readily understood that whenever this scale is used in conjunction with the A and S scales the answer must be divided by 10.

A similar method is used to find the values of expressions such as x sin y, or, x tan y.

E.g. Find the value of 6 sin 20°

Set sin 20° to R.H. index window, the value of sin 20° can be read on B scale over R.H.1 on A scale. Keeping the slide in the same position the value of 6 sin 20° can be read on B scale below 6 on A scale ∴ 6 sin 20° = 2·05. Tangent values can be read in the same manner.

For practical trigonometrical calculations the user should study the following section on Survey in this book

THE "TECHNICAL" SURVEYOR'S SLIDE RULE

This is especially designed for use in land survey, aerial and sea navigation. In addition to the scales on the "Technical" Standard Slide Rule, the Surveyors' rule is equipped with the CR scale in the centre of the slide and on the reverse side the complementary values of the sines and tangents called respectively the Cos and Cot scales. These additional scales are graduated in red for easy reference. This rule is also equipped with two linear scales extending from the sides of the rule, one to facilitate measuring distances on the 1/1000000 and 1/500000 air maps, and the other a practical ordnance scale for measuring distances on maps of 1/250,000.

THE GAUGE MARKS

In addition to the mark π the Surveyor's Slide Rule has the following points marked on the face of the rule.

The T and T′ mark is set at 1146 on the A and B scales only. The value of $\frac{1}{1146} = \frac{\pi \times 3}{180 \times 60}$.

This constant is useful for gunnery calculations where angles involved are less than 20°, in which case the circular measure and the tangent of the angle may be regarded as equal.

In using the T or T′ mark it is necessary that the reading of the angle is taken in minutes, the subsidiary base in feet and the base in yards.

E.g. To find the base given the sub base (b) in feet and the angle (a) in minutes.

Set b on B scale under a on A scale and over T (or T′) on B scale read the length of the base in yards on the A scale. To determine the position of the decimal point, it should be noted that when the scales are set in the L.H. A scale the reading should be taken off the T mark, and if the R.H. is used the reading is taken off the T′ mark.

The marks Q′, Q″ and $Q_{\prime\prime}$ are found on the D scale only, and are employed to convert angles into circular measure, or *vice versa*, and also to determine the functions of small angles.

The Q′ mark is positioned at 3437·74 and gives the number of minutes in a radian. The value is equivalent to $\frac{180 \times 60}{\pi}$

The Q″ mark expresses the number of seconds in a radian, and is set at 206265 which equals $\frac{180 \times 60 \times 60}{\pi}$.

The $Q_{\prime\prime}$ mark is positioned at 636620 which represents the value of $\frac{200 \times 100 \times 100}{\pi}$ and expresses the number of seconds in a radian in the French decimal system. The latter is useful when the newer graduation of the circle is employed.

CONVERSION OF LINEAR UNITS

On the reverse of the centre slide there will be seen two red gauge points marked F and Y. These are used in conjunction with the A scale for the

conversion of yards or feet to metres and *vice versa*. The pivot for these calculations is the line at sin 90°.

Thus to convert metres to yards, place sin 90° under the number on A scale and read the number of yards over Y on A scale

E.g.	Scale A	9·14	10
	Scale S	90°	Y

To convert yards to metres place Y under the number on A scale and read the corresponding number of metres above sin 90°.

By using the F mark feet can be converted to yards or metres, and *vice versa*. The following illustrations clarify the method.

To convert feet to yards

Scale A	*n* yards	*n* feet
Scale S	F	90°

To convert feet to metres

Scale A	*n* metres	*n* feet
Scale S	F	Y

THE CR SCALE

The reciprocals of any numbers are obtained by reading the number on C to Cr or *vice versa* thus :—

5 in C projects into ·2 on Cr.

4 in C projects into ·25 on CR.

E.g., $\frac{1}{71} = \cdot 0141.$

The use of this scale also simplifies evaluating expressions in the form of $a \times b \times c$ or $\frac{a}{b \times c.}$ Thus let $a = 3{\cdot}6$; $b = 4{\cdot}4$; $c = 1{\cdot}5$.

Example 1.—Set the cursor on 3·6 in D scale and bring 4·4 in CR into coincidence, the result 23·76 can be read in D under 1·5 in C.

Example 2.—Set 3·6 in D to coincide with 1·5 in C. The result ·545 can be read in D opposite 4·4 in CR.

The position of the decimal point is usually obvious from the nature of the calculation.

THE TRIGONOMETRICAL SCALES.

It is assumed that the users of the " Technical " Surveyors Slide Rule are familiar with the general use of the S, T, and ST scales. We now come to the method of obtaining the values of Cosines, Cotangents and the Tangent values which are greater than those graduated on the slide.

Cosines from 0° to 89° 25′ 35″ are read on the Cos scale from right to left in conjunction with A and B scales. The values on the left hand read from ·01 to 0·1 (Cos. 89° 25′ to Cos 84° 17′) while the R.H. reads from 0·1 to 1·0 (cos 84° 17′ to 0°).

E.g., Cos 26° = ·8988. Set 26° on Cos scale to R.H. index window and read below the R.H. 1 on A scale the answer on B scale.

Tangents of angles greater than 45° are set to the Cot scale and the value read by projecting Cursor into CR scale.

E.g., Tan. 73° = 3·271 Set Cot 73° to R.H. index window, and read the result by projecting R.H. 1 of D scale into the CR scale.

Cotangents. The values of Cotangents 45° to 84°17′ are obtained by setting the angle to either R.H. or L.H. index window and reading the result on C opposite 1 in D.

E.g., Cot 68° = ·404, Set 68° on Cot scale to R.H. index window and read the result on C scale over R.H. 1 on D scale.

For cotangents of less than 45° set the angle to R.H. or L.H. back window and read the result opposite R.H. or L.H. 1 on D on the CR scale.

E.g., Cot 27° = 1·96, Set 27° on T scale to R.H. index window and read the result by projecting the R.H. 1 on D scale into CR scale.

Cosecants and Secants are obtained as the reciprocals of the Sine and Cosine respectively.

CHORDS

To obtain the chord of an arc given an included angle and the radius. Set AB, C and D scales into coincidence, and reverse the rule to bring *one-half* of the angle on the sine scale into line with the index mark on the back window. The chord can then be read on the B scale coincident with *twice* the radius on the A scale.

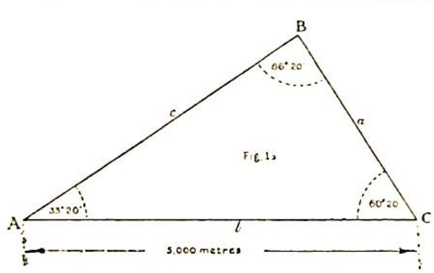
Example.—To find the chord of an arc given angle 20° radius 24″ reverse rule and set 10° on the S scale (being half the angle) in line with index at R.H. window. The length of the chord of the arc can be read on B scale directly below 48 on the A scale = 8·34″.

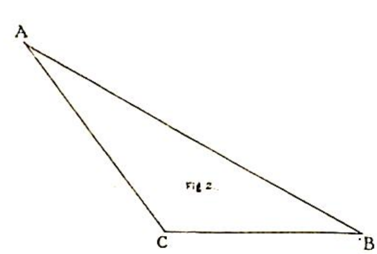
THE SOLUTION OF TRIANGLES.

Example 1.—To solve a triangle given three angles and one side. Place the cursor on the number of the known side on the A scale, then set the angle opposite that side into coincidence using the S scale. The respective lengths of the other two sides can then be read on the A scale opposite the respective angles.

Thus—(see fig. 1)

Set 86°20′ on the S scale under 50 on the A scale, and over angle 60° 20′





read the length of c = 4,350 metres, and over angle 33° 20′ read the length of a = 2,760 metres.

Example 2.—Given two sides and one angle it is required to find the ABC angle and distance from A to B (see fig. 2). Let the two known sides be AC and CB. AC = 8,000 yards ; CB = 5,000 yards. Angle ACB = 140°.

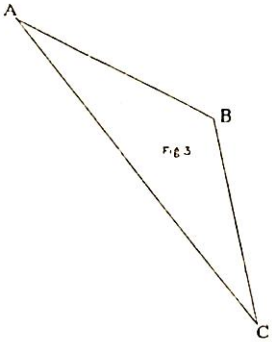
Since the three angles of a triangle are equal to 180°, then CAB + ABC = 180° − 140° = 40° The S scale should now be set in a position that the sum of two angles at points 80 and 50 on the A scale are together equal to 40°.

The slide rule should appear thus ·—

Scale A	50	80	100	
Scale S	15°10′	24°50′		40°

The angle ABC can be read under 80 = 24°50′.

Over angle ACB 40° read the length A to B = 12,250 yds. It will be noted that the 40° mark extends beyond the 100, and it is therefore necessary to cross the slide to obtain the result, alternatively the 8 and 5 (L.H.) of the A scale should be used, the position of the decimal point should be fixed accordingly.



Example 3.—Sometimes it is found that one of the unknown angles CAB or ABC is an obtuse angle, that is greater than 90°, see fig. 3. In this case it is not possible to obtain on a slide rule, the two angles that added together are equal to 180°—ACB. Therefore it is necessary to find two angles, which on subtracting one from the other the difference would be equal to the angle ACB.

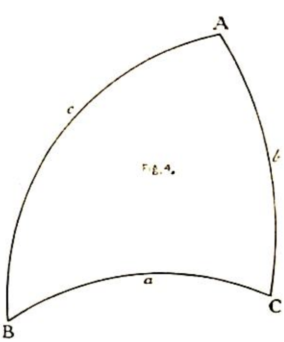
Then the angle opposite the greater side is to be subtracted from 180°. In order to carry out the following exercise it will be necessary to use two cursors.

E.g, Let AC = 3,000 metres; CB = 1,200 metres; angle ACB = 23° (then the angle ABC is greater than 90°). Place one cursor over 3,000 and the other over 1,200 on the A scale. The slide should now be manipulated so that the *difference* of the angles coincident with the cursor lines is 23°. It will be found that under 3,000 on A scale the angle is 36°55′, ∴ angle ACB + CAB = 36°55′, and as AC is the longer side then the angle ABC is equal to 180° − 36°55′ = 143°5′. By projecting 23° into the A scale the length of AB is obtained = 1955 metres.

SOLUTION OF SPHERICAL TRIANGLES

The solution of spherical triangles can be readily attained by the use of the sine formulae :—

$$\frac{\text{Sin B}}{\sin\ b} = \frac{\text{Sin C}}{\sin\ c}: \quad \text{SIN B} = \frac{\text{Sin C} \sin b}{\sin c}$$



Example 1.—It is required to find the value of the angle B in a spherical triangle where the side b = 58°30′; c = 62° and the angle C = 40°25′, see fig. 4. Therefore by use of the above formulae

$$\text{SIN B} = \frac{\text{SIN } 40°25' \quad \text{Sin } 58°30'}{\text{Sin } 62°}$$

The following values should be jotted down on a piece of paper :—

Set 40°25′ S to R.H. index window reverse rule and read below 100 on A scale the value ·648 on B scale.

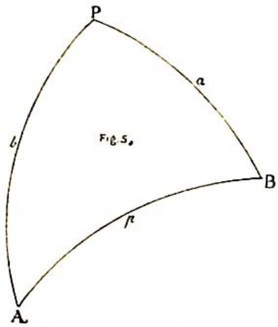
Set 58°30′ in the same manner and read the value ·852.

Set 62° in the same manner and read the value ·883.

Now using the face of the rule set 1 on B scale under ·648 on the A scale,

then set cursor to ·852 on B scale reading ·55 on A scale, without moving cursor set ·883 on B scale into coincidence the value of Sin B can then be read directly over 1 on B on the A scale ∴ Sin B = ·62.

To find the angle set 62 on B below 100 on A scale and read the angle 38°30′ on the R.H. index window. ∴ Angle B = 38°30′



Example 2.—In the spherical triangle P.A.B. (fig. 5) it is required to find the length of the side p in minutes of arc, when P = 25°46′, a = 50°32′ and b = 112°. In this instance we use the following Cosine formulae.

Cos p = Cos a, Cos b. + sin a, sin b, Cos P.

∴ Cos p = Cos 50°42′, Cos 112° + Sin 50°42′, Sin 112°, Cos 25°46′.

= Cos 50°42′ (—Cos 68°) + Sin 50°42′, Sin 68°, Cos 25°46′.

With use of the back index window read off the natural sin and cos values of a, b, and P. It will be found: a = 50°42′ value sin a = ·774.

b = 112° using complementary 68° read value sin b = .927.

Using Cos scale P = 25°46′ read value Cos P = ·900.

a = 50°42′. Cos A = ·633 ; b = 112°, Cos b = — ·375.

Using the face of the rule bring L.H. 1 of B to ·774 on A scale and over ·927 on B scale read ·718 on A scale. Set cursor to ·718 on A scale then move slide to bring ·9 on B coincident to 1 on A scale, and under cursor read ·647 on B scale. Now set R.H.1 of B scale below ·633 on A scale and over ·375 on B scale read —·238 on A scale. Note that this value is minus as Cos 112° is—·375. By adding ·647 to—·238 we have the nat. cos p = ·409.

Now set R.H. 1 on A over ·409 on B scale and read 65°50′ on Cos scale under index window at back. ∴ p = 65°50′.

NAVIGATIONAL NOTES.

Ratio Proportion and Conversion Factors.

With the aid of the slide rule the conversion of different units of measure, length, speed, etc., can easily be accomplished.

If the scales A and B or C and D are set in any relative position the ratio of cursor readings is constant to that setting. Hence problems of wind drift, speed or time can be solved by setting the A and B scales or C and D scales in the relative position to the factors. The following table will demonstrate the method.

Constant	Slide Rule Setting		Read on Scale C	Scale D
1 knot = 1·15 m.p.h.	L.H. 1 on C	1·15 on D	Knots	m.p.h.
Kilom=·54 Naut. miles	R.H. 1 on C	5·4 on D	Kilometres	Naut. Miles
1 litre = ·22 gallons	2·2 on C	L.H. 1 on D	Gallons	Litres
1 litre = 1·76 pints (approx.)	L.H. 1 on C (nearly)	1·76 on D	Litres	Pints
1 m.p.h.=1·61 k.p.h.	1·61 on C	L.H. 1 on D	k.p.h.	m.p.h.

Almost all other ratio and proportion factors can be converted speedily by this method.

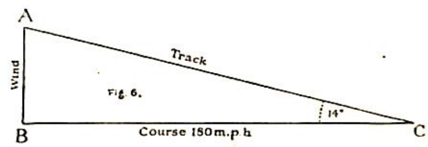
PROBLEMS OF GROUND SPEED, COURSE TO STEER, Etc.

To find the course to steer and ground speed along a given track use the sine and A scales.

Set the angle on the bow or quarter of the track to which the wind is blowing, under the Air Speed. Then under the wind speed the drift angle can be read. Add or subtract the drift angle to the track (a head wind is subtracted and a tail wind added) this will give the course to steer.

• Above the course to steer on the Sine scale the ground speed can be read directly on the A scale. When wind is at right angles use TAN scale as example 1.

Example 1.—An aircraft is set due East at 180 m.p.h. with wind blowing due North (see fig 6). Drift is 14°. It is required to find the wind speed and ground speed.



The difference between the wind direction and set course in the illustration is equal to 90°.

$$\therefore \quad \frac{\text{Wind}}{\text{Course}} = \text{TAN C} = 14°.$$

To find the wind speed set Tan 14° to L.H. index window, the wind speed can then be read on the C scale over 180 on the D scale.

Wind speed = 44·9 m.p.h.

The ground speed required is along track this is equal to secant *a* 14°. To find the ground speed set cos 14° to the R.H. index window and over 18 on B scale read the answer on A scale ; ground speed = 185·6 m.p.h.

Example 2.—It is required to find the course to steer and ground speed from the given data :—Air speed 120 m.p.h. ; Track 050° True ; Wind velocity 25 m.p.h. from 090°T (40° on the bow).

Reverse the slide and bring the S scale to the top. Under 120 on A scale set 40° S scale and under 25 read 7°40′ (drift angle). Now subtract the drift angle from the track (50°—7°40′ = 42°20′) over 42°20′ read the ground speed = 126.

∴ Course to steer = 42°20′
Ground speed = 126 m.p.h.

To find the *Wind Velocity* when the Track, Ground Speed, Course and Air Speed are known.

Set the cursors into coincidence with the known Air Speed and Ground Speed on the A scale. Now adjust the slide so that the number of degrees projected beneath the Air Speed and Ground Speed are equal to the Drift Angle (the difference between course and track). Then above the Drift Angle on the sine scale read the Wind Speed on Scale A, and below the Air Speed read the wind direction as an angle on bow or quarter of the track.

The wind direction as an angle on the bow or quarter of the course will be found below the ground speed. If the ground speed is less then the air speed, the angle is on the bow ; if the air speed is less than the ground speed then the angle is on the quarter.

Example.—It is required to find the Wind Speed, Wind Direction and Drift from the following :—

Given :—Course 140°T	Air speed 140 m.p.h.
Track 147°T	Ground speed 120 m.p.h.

Reverse the slide and bring the S scale to the face of the rule. Set the cursor to 120 on the A scale and place a light pencil mark on the cursor over 140. The slide is now to be adjusted until the difference of the angles coincident to the cursor lines is equal to 7°. In this example 35° and 42° will be found to coincide with the cursor lines.

The wind speed can be read on the A scale above 7° on the Sine scale = 25·5 m.p.h. The wind direction is 42° on the bow of the track and is therefore 105°T, as the drift is to the starboard.

GROUND SPEED, Etc. FOR HEIGHTS

The following method will be found useful when it is desired to find the ground speed while flying over the sea.

The drift angle is usually found by a Drift Sight on back-bearings of an object dropped from the aircraft, but the determination of the ground speed presents a little more difficulty. It is established that the difference between the wind direction at the surface and the wind direction at a reasonable height is nearly constant with any changes of wind direction at the surface.

This difference can be obtained from meteorological information at the starting point, and it can be applied to the surface wind direction obtained by taking bearings of the wind lanes on the surface of the sea during flight.

A fairly accurate forecast of the direction of the upper wind may be obtained from the meteorologist and may be checked by observing the movement of cloud shadows on the surface of the sea.

Example.—Given the Course and Air Speed, Drift Angle and Wind Direction, it is required to find the Wind Speed and Ground Speed.

Air speed 120 knots ; course 14°T ; Track 145°T ; Drift 5° to starboard ; Wind direction 345°T.

The difference between Track and Wind Direction is 200°. The wind angle is therefore 20° on the quarter and is a tail wind. Now 20° when added to 5° (Drift) will give the angle to find the Ground Speed.

Reverse the slide bringing the sine, etc. scales to the face of the rule. Then set 20° on the sine scale under 12 or 120 on the A scale, and over 5° read the Wind speed = 30·6. The ground speed is read on the A scale above 25° = 148.

∴ Wind speed = 30·6 knots.
Ground speed = 148 knots.

RHUMB LINE TRACK AND DISTANCE

With the aid of the slide rule the Rhumb Line Track and Distance can readily be obtained by use of the following formulae :—

$$\text{Departure} = \text{D Long} \times \text{Cos. Mid Lat.}$$

$$\text{Tan Track} = \frac{\text{Departure}}{\text{D. Lat.}}$$

$$\text{Distance} = \frac{\text{Departure}}{\text{Sin Track}} \quad \left(\text{or} \quad \frac{\text{D. Lat.}}{\text{Cos. Track}}\right)$$

Example.—It is required to find the Rhumb Line Track and Distance from Ostend (Belgium) to Esbjerg (Denmark).

From Ostend	Lat. 50°12′ N.	Long 2°09′E.
to Esbjerg	Lat. 55°6′ N.	Long. 8°20′E.
	D. Lat. 4°54′ N. = 294′	D. Long. 6°11′E. = 371′

Mid. Lat. 52°39′ N.

To find the Departure. Set Cos 52°39′ to R.H. index window and read on B scale under R.H. 1 on A scale 0·61 (Cos Mid. Lat.). The Departure can now be read on the same setting of the slide, on the B scale below 371 on A scale. ∴ Departure = 226′.

To find the True Track. Set the smaller value of Dep. and D. Lat. on C scale, and the larger on D scale. By this method over 294 on scale D set 226 on scale C. The true track can be read on the Tan scale coincident to R.H. index window.

True track = 37°25′.

To find the Rhumb Line Distance. Set 37°25′ on the sine scale to R.H. index window, and above 226 on B scale read the result (372) on A scale.

∴ Rhumb line distance = 372 miles.

CORRECTION OF REFRACTION

For altitudes above 8° the cotangent of the angle should be multiplied by ·96 (or ·96 may be divided by the tangent of the angle).

Example.—To find the correction of Refraction for an altitude of 40°.

Set Tan 40° to R.H. index window and below ·96 on C scale read the correction on D scale = 1·14.

∴ Correction is 1·14 minutes (minus) to apparent horizon.

CORRECTION FOR DIP OF SEA HORIZON.

The formulae for the dip is given by $\sqrt{\text{Height}} \times \cdot 98$

Example.—It is required to find the dip correction when flying at 750 feet.

Set the L.H. 1 on scale B under 750 on scale A, and under ·98 on C scale read the answer on D scale.

The correction for dip is therefore 26·83 minutes (minus) to the altitude observed.

THE TECHNICAL AERONAUTICAL SLIDE RULE

This is equipped with thirteen logarithmic scales. On the face of the rule there are ten scales.

The use of the A, B, C, D, log-log (M and N) and CR scales have been explained earlier in this book, as those on the reverse of the slide carrying the Sine Tangent and ST scales.

THE P AND P[1] SCALES

These scales read from π to π, and are used in conjunction with the C and D scales. Thus any one calculation involving the use of π can be obtained without setting the slide.

E.g. It is required to find the diameter of a circle 15·71 in circumference. $\therefore$ Diameter $= \frac{15\cdot17}{\pi}$ Set cursor to 15·71 in P scale and read the result (5) in D scale.

Similarly if the diameter is known, by projecting the number into P scale this is automatically multiplied by π. *E.g.* It is required to find the circumference of a circle 8·3 in diameter. Set the cursor to 8·3 on D scale and read the result on P scale.

Circumference = 26·08 (approximately).

When evaluating an expression where π appears in the denominater or numerator the π should be ignored, and the result read in the P or D scale.

It will be generally noticed that when making a calculation on the C and D scales, it is often necessary to cross the slide to read the result. Since the P and P^1 scales represent an extended duplication of the C and D scales, they can be used in conjunction when evaluating an expression combining multiplication and division, and it will be found that the result can always be obtained on the rule without crossing the slide.

Example.—(*a*) Multiply 3·4 by 8·2. Set 1 on C scale coincident to 3·4 on D scale the result 27·88 can be read on P scale over 8·2 on P^1 scale.

(*b*) $$\frac{\pi\,(2\cdot2 \times 5\cdot5 \times 6\cdot2)}{3\cdot1}$$

Set L.H.1 on C scale over 2·2 on D scale, move cursor to 5·5 on P^1 scale, then move slide to bring 1 on P^1 into coincidence with cursor line, now adjust cursor to 6·2 on P^1 scale and bring 3·1 on P^1 scale into cursor line. The result 76) can now be read by projecting R.H. 1 on D scale into P scale.

THE K SCALE

The K scale represents the cube of the numbers on the D scale and can be read off directly with the aid of the cursor. *E.g.* To find the cube of 6, set the cursor at 6 on D scale, the result can be read directly on the K scale · $6^3 = 216$.

The cube root of any number can be readily obtained by selecting the number on the K scale and the result read on D scale. *E.g.* $\sqrt[3]{125} = 5$.

Where an expression of the form $(a \times b \times c \times d)^3$ is to be evaluated this may be done in one series of operations by using the P or CR scales and reading the final result on K.

E.g. $(3\cdot2 \times 3\cdot5 \times \cdot25 \times 2\cdot5)^3$

Set cursor to 3·2 on D scale and bring 3·5 on CR into coincidence, move cursor to ·25 on C scale and bring 2·5 on CR into coincidence. The result can now be read by projecting the R.H. 1 on C scale into K scale. Answer = 343.

THE GAUGE MARKS

The gauge marks C, C′ and M have been explained in the Standard Section of this book, while the value of the marks Q′, Q″ and $Q_{\prime\prime}$ have been dealt with in the Survey Section.

On the face of the rule there will be seen the additional gauge points marked W, N and V. These are used in conjunction with the C and D and P and P^1 scales for the calculation of Electrical Formulae.

The W Mark is set in position at ·746-1 on the C, D, P and P^1 scales so that

if the L.H. index is set to Kilowatts the horse power can be read opposite the gauge mark W on the respective scale.

E.g. Convert 61 h.p. to Kilowatts.

Set R.H. 1 in C over 61 on D and read the result by projecting W on C into D = 45·5 Kilowatts. It will be noticed that the same result is obtained by projecting W on P^1 into P.

The gauge point N represents the reciprocal of the W mark and is positioned at 1·3404 on the P^1 scale. Thus, to convert Kilowatts to horse power set 1 on C, or, P^1, to Kilowatts and read the horse power by projecting N into P scale.

E.g. Convert 61·5 Kilowatts to horse power. Set 1 on P^1 scale below 61·5 on P scale and read the result opposite N on P scale = 82·5 h.p.

A similar method can be used to calculate the efficiency of a dynamo and motor.

E.g. (1) Calculate the efficiency of a dynamo. Output 21 KW. for 40 h.p. Set 40 on C coincident with 21 on D. Read the result on P scale opposite N. Efficiency 70·5 per cent.

E.g. (2) A motor develops 138 h.p. for 120 KW. Calculate the efficiency. Set 120 on C over 158 on D scale and read the result by projecting W on C into D scale. Efficiency = 86·4 per cent. Similarly the result can be obtained by projecting W on scale P^1 into P scale.

The V Mark is set on each end of the P^1 scale, and is used to calculate the drop in pressure along a copper conductor. The resistance mark is placed at 30·5 and set for 60°F. A correction of temperature can be made by using the formulae $C^\circ = 5/9\,(F^\circ - 32^\circ)$; $F^\circ = 9/5\,(C^\circ + 32^\circ)$.

Since the resistance of a conductor depends upon its length, cross-sectional area and temperature, the drop in pressure can be obtained from the following formulae.

$$\text{Drop} = \frac{\text{Amperes and length}}{\text{cross sectional area}} \times \text{resistance term (30·5).}$$

Example.—It is required to find the voltage drop of a copper conductor given length = 400 yards ; area of cross section = 22,000 circular mils ; current 14·5 amps. Set 1 on C scale over 14·5 on D scale and move cursor to 4 on C. Move 22 on C scale to cursor, project Mark V on P^1 to P scale and read the result (8·04 volts).

CALCULATIONS OF SHEAR STRESS ON A SOLID SHAFT

Special Note—

[For the benefit of those whose experience of the Slide Rule is rather limited the computations in the following examples have been made in the simplest manner, *i.e.* by the successive applications of the appropriate numerators and denominators. Those more familiar with the scales will soon find that the calculations can be shortened in many cases by using the CR, P and P^1 Scales in conjunction with the A, B, C and D scales as appropriate to the individual circumstances].

A solid shaft of 8″ diameter is subjected to a torque of 12·5 ton/ft. What is the extreme shear stress introduced.

(1) Formulae :—$T = f(s) \times \dfrac{\pi\ D^3}{16}$

Where T = Torque, $f(s)$ = shear stress and D = Diameter of shaft.

$$\therefore \quad f(s) = \frac{16T}{\pi D^3}$$

$$\therefore \quad fs = \frac{12{\cdot}5 \times 2240 \times 12 \times 16}{\pi \times 8^3}$$

(Note reductions of tons/ft. to lbs./ins. in upper line)

First set cursor over 8 in scale D and under cursor read off in scale K, 512 (*i.e.* $8^3 = 512$).

Now bring cursor over 12·5 in scale P, bringing 512 on scale C under cursor, move cursor to 224 on scale C, bring L.H. 1 on scale C to cursor, move cursor to 16 on scale C bring L.H. 1 on scale C to cursor and under 12·5 on scale C read 334 on scale D.

$$\therefore \quad f(s) \quad \text{Shear stress} = 3{,}340 \text{ lbs./ins.}$$

(2) Finding the modulus of Elasticity and the deflection of a specimen of timber, arranged as a simply supported beam.

A specimen of spruce, rectangular in section 2″ wide and 1″ deep is arranged as a simply supported beam on 2 ft. span with the load centrally applied. It deflects ·0576 ins. under a 40 lbs. load. What is the value of E (the modulus of Elasticity) of the material? What will be the deflections when the load has been increased, until the bending stress equals 1,200 lbs. per sq. in?

Formulae 1.

(*a*) Deflection at centre : $(y) \quad = \quad \frac{Wl^3}{48\,E\,I}$

Where W = load, l = length of beam, E = Modulus of Elasticity, I = Moment of Inertia and y = Deflection.

Formulae 2. Where $E = \frac{Wl^3}{48\,Iy}$

$$\text{Now } I = \frac{bd^3}{2} \begin{pmatrix}\text{where } b = \text{breadth of beam} \\ \text{and } d = \text{depth of beam}\end{pmatrix} = \frac{2 \times 1^3}{12} = \frac{1}{6}$$

$$\therefore \quad \text{(from formulae 2) } E = \frac{40 \times 24^3 \times 6}{48 \times 0{\cdot}0576 \times 1}$$

First move cursor to 24 on D scale and under cursor, on K scale read 13820 (*i.e.* $24^3 = 13820$) note this down.

Now set 48 on C scale over 40 on D scale, move cursor to 1382 on C scale bring L.H. 1 on C scale to cursor, move cursor to 6 on C scale bring 576 on C scale to cursor and under L.H. 1 on C scale read on D scale.

1,200,000 lbs. per sq. inch = E.

(*b*) Now when f = 1,200 lb./sq. in.

$$M = fZ \begin{pmatrix}\text{Where } M = \text{Bending Moment, } f = \text{Bending stress} \\ Wl = \text{weight} \times \text{length of arm and } Z = \text{Modulus of Section}\end{pmatrix}$$

$$= 1200 \times \frac{1}{6 \times \frac{1}{2}}$$

$$= 400 \text{ lbs. ins.}$$

$$\text{But } M = \frac{Wl}{4} = 6W$$

$$\therefore \quad W = \frac{400}{6} \text{ lbs.}$$

Now Deflection at centre due to this load $= \frac{Wl^3}{48\,EI}$ (from (1))

$$= \frac{400 \times 24^3 \times 6}{6 \times 48 \times 1{\cdot}2 \times 10^6 \times 1}$$

(Note that the 6 on the upper line cancels with the 6 on the lower line, and that the 1 on the lower line may be neglected).

Now set 48 on scale C over 40 on scale D, bring cursor to 1382 on scale C, set 12 on scale C to cursor and under R.H. 1 of scale C read on scale D.

0·096 inch = Deflection.

(3) ESTIMATION OF SAFE LOAD ON A STRUT BY ELUER'S FORMULAE.

A solid wrought iron strut 2″ in diameter and 6 ft. long with both ends pin-jointed is subject to an axial compressive load. Taking a safety factor of 3 and assuming E = 12,500 tons per sq. in., estimate the safe load for this strut by Eulers Formulae. Now where P = load, E = Modulus of Elasticity and I = Moment of Inertion and l = length.

Then by Euler's Formulae $P = \frac{\pi^2\,EI}{l^2}$

Now for a circular section,

$$I = \frac{\pi}{64}\,D^4 = \frac{\pi}{64}\,2^4 = \frac{\pi}{4}$$

$$\therefore \quad P = \frac{\pi^2 \times 12500 \times \pi}{(6 \times 12)^2 \times 4}$$

First bring the cursor to π on scale C and under cursor on scale K read 31 ($\pi^3 = 31$). Note this down.

Now set L.H. 1 of scale C to 31 on scale D, move cursor to 12·5 on scale C, set 72 ($= 6 \times 12$) on scale C to cursor, move cursor to R.H. 1 of scale C bring 4 on scale C to cursor and under L.H. 1 of scale C read on scale D :—

18·69 tons = P.

Now 18·69 tons = maximum load, and, as the safety factor is 3, then safe load $= \frac{18{\cdot}69}{3} = 6{\cdot}23$ tons.

(4) FINDING THE DRAG ON AN AEROPLANE WINDSCREEN OF FLAT SURFACE.

An aeroplane windscreen is a flat plate 15 ins. × 8 ins. Find its drag when travelling at 60 m.p.h. at sea level, given $C_R = 1{\cdot}24$ and $\rho = .00238$.

Now where R = Drag, C_R = Coefficient of resistance, ρ = fluid density (in slugs per cubic foot), S = surface (in sq. ft.) and V = Velocity.

Then $R = C_R\,\frac{1}{2}\,\rho\,S\,V^2$.

$$\therefore \quad R = 1{\cdot}24 \times \frac{{\cdot}00238}{2} \times \frac{15}{12} \times \frac{8}{12} \times 88^2$$

(Note that length and breadth are brought to feet, and speed (60 m.p.h.) is shown as 88 ft. per second).

First bring cursor to 88 on scale D and under cursor on scale A read 7744 ($88^2 = 7744$). Note this down.

Now set L.H. 1 on scale C to 1·25 on scale D move cursor to 2·38 on scale C,

set 2 on scale C to cursor, move cursor to 15 on C scale, set 12 on C scale to cursor, and over 8 on P^1 scale read 14·68 on scale P. (Note here the useful extension of the C and D scales in P and P^1 scales). Now bring 12 on scale C over 14·68 on scale D, move cursor to 7·75 on scale C and under cursor read on D scale

$$9{\cdot}45 \text{ lbs.} = R = \text{Drag of screen.}$$

(5) FINDING COEFFICIENTS OF LIFT AND DRAG AND LOWEST SPEED OF LEVEL FLIGHT OF AN AEROPLANE.

The lift on an aeroplane wing is 2250 lbs. and the drag is 98 lbs. If the wing area is 180 sq. ft. and the flying speed is 150 m.p.h. find the lift and drag coefficients. If maximum lift coefficient is 1·5, what is the lowest speed of flight in m.p.h. (given that $\rho = {\cdot}00238$ slugs per cu. ft.)

Here C_L = Coefficient of lift, C_D = Coefficient of drag, ρ = fluid density of the air, L = lift in lbs., S = wing area in sq. ft., V = speed of flight in ft. per second.

(Note that $\frac{88}{60}$ is factor to convert m.p.h. to ft. per sec.).

$$\text{Coefficient of lift } C_L = \frac{L}{\frac{1}{2}\rho SV^2}$$

$$= \frac{2250}{\frac{{\cdot}00238}{2} \times 180 \times 150^2 \times \left(\frac{88}{60}\right)^2}$$

$$= \frac{1}{{\cdot}00119 \times 10{\cdot} \times \left(\frac{88}{60}\right)^2}$$

(Note cancellation).

Set 60 on scale C over 88 on scale D, move cursor to L.H. 1 on scale C and read, on scale A under cursor, 2·15 (*i.e.* $\left(\frac{88}{60}\right)^2 = 2{\cdot}15$).

Now set L.H. 1 on scale C to 2·15 on scale D move cursor to 18 on scale C bring L.H. 1 on scale C to cursor and move cursor to 119 on scale C. Now set L.H. 1 on scale C to L.H. 1 of scale D and project cursor to read on CR scale. ${\cdot}2175 = C_L$ = Lift Coefficient.

(*b*) $$\text{Drag coefficient } C_D = \frac{D}{\frac{1}{2}\rho SV^2}$$

where D = Wing drag in lbs. $\times \frac{C_L}{L}$ (see No. 5 (*a*)).

$\therefore \quad C_D = 98 \times \frac{0{\cdot}217}{2250}$ (as $\frac{1}{2}\rho SV^2$ is constant here it need not be taken into account).

Set 225 on scale C over 217 on scale D and under 98 on scale C read 0·00945 on scale D.

$\therefore \quad C_D = 0{\cdot}00945$ = Coefficient of Drag.

(*c*) Lowest speed of flight at maximum of lift coefficient will be

$$\sqrt{\frac{L}{\frac{1}{2}\rho S C_L \max.}}$$

Note C_L max. is given at 1·5 in the question.

$$= \sqrt{\frac{2250}{\cdot 00119 \times 180 \times 1\cdot 5}}$$

Set L.H. 1 of scale C to 119 on scale D, move cursor to 18 on scale C bring L.H. 1 on scale C to cursor, move cursor to 1·5 on scale C, bring 225 on scale C to cursor and over R.H. 1 of scale D read 7,000 on scale C. Now move cursor to 70 on right hand part of scale A and under cursor on scale D read 83·6 (*i.e.* $\sqrt{7000} = 83\cdot 6$).

$\therefore$ Lowest speed = 83·6 ft. per second.

Now multiply this by $\frac{60}{88}$ to bring to m.p.h.

Set 88 on scale C over 83·6 on scale D and under 60 on scale C read 57 m.p.h. on scale D.

$\therefore$ 57 m.p.h. = lowest speed of flight.

(6) FINDING SPEED OF LEVEL FLIGHT OF A MONOPLANE AT MAXIMUM $\frac{L}{D}$ WHERE WING LOAD IS KNOWN.

The maximum value of the lift over drag ratio, of a monoplane occurs when the lift coefficient, C_L, is ·826. Determine at what speed the plane can fly in level flight with a wing loading of 12 lbs. per sq. ft. at the maximum value of $\frac{L}{D}$

The speed in level flight for a wing loading of 12 lbs. per sq. ft. is

$$V = \sqrt{\frac{W}{\frac{1}{2}\rho \times C_L}}$$

Where V = speed in ft. per sec.; W = Wing load in lbs. per sq. ft.; ρ = fluid density of air and C_L = coefficient of lift.

$$\therefore \quad V = \sqrt{\frac{12}{\cdot 00119 \times \cdot 826}}$$

Set 826 on scale C over 12 on scale D, bring cursor to R.H. 1 of scale C, set 119 on scale C to cursor and under L.H. 1 of scale C read 12,200. Now bring cursor to 1·22 on left hand part of scale A under cursor on scale D read 110·5 ($110\cdot 5 = \sqrt{12{,}000}$) = 110·5 ft. per sec.

Now multiply 110·5 by $\frac{60}{88}$ as in No. 5 (c).

This gives 75·4 m.p.h. = lowest speed in level flight.

(7) FINDING THE TAIL LIFT OF A MONOPLANE.

A monoplane of weight $3\frac{1}{2}$ tons has a wing chord of 10 ft. and wing span of 70 ft. and is gliding at 140 m.p.h. Calculate the lift on the tail plane, given that the wing moment coefficient $\overline{C}_M$ about the Centre of Gravity (CG) is $-0\cdot 032$ and the arm of the tail moment ($= l$ = length of tail arm) is 28 ft.

Given M = moment of tail lift, $\overline{C}_M$ = moment of wing about centre of gravity of plane (CG), S = Wing surface, ρ = fluid density of the air and C = chord of aeroplane wing. The chord of a wing = breadth (straight line between leading edge and rear edge) of wing.

$$Formulae. — M = \overline{C}_M \frac{\rho}{2} S V^2 C.$$

$$= -0{\cdot}032 \times 0{\cdot}0119 \times 700 \times (140^2 \times 2{\cdot}149) \times 10.$$

Note that $2{\cdot}149 = \frac{88}{60}^2$ = (conversion factor)^2 to convert m.p.h. to ft. per sec.

Set cursor to 14 on scale D and under cursor on scale A read 196. (*i.e.* $14^2 = 196$). Note this down.

Now set L.H. 1 of scale C to 196 on scale D move cursor to 2·15 on scale C, bring R.H.1 of scale C to cursor, move cursor to 7 on scale C, bring L.H. 1 of scale C to cursor, then move cursor to 119 on scale C, bring R.H. 1 of C to cursor and under 32 on scale C read on scale D.

$$-11{,}200 \text{ ft. lbs.} = M = \text{lift moment of tail.}$$

$$\text{Hence tail lift } P = \frac{M}{l} = \frac{\text{Tail lift moment.}}{\text{length of tail moment arm.}}$$

$$= \frac{-11200}{28} = -400 \text{ lbs.}$$

$$\text{Hence } P = 400 \text{ lbs. down.}$$

(8) FINDING ANGLE OF BANK AND LOAD FACTOR DURING TURNING.

An aeroplane of 2550 lbs. weight has a wing area of 125 sq. ft. Area for which C_L max. (maximum Lift Coefficient) is 1·2. Find the maximum angle of bank without sideslip which can take place at 170 m.p.h. What is the load factor during the turn.

$$\text{Wing Lift } L = C_L \text{ max. } \tfrac{1}{2} \rho S V^2$$

(The above symbols have the same meanings as they had in Example 7 and previous examples).

$$\therefore\ L = 1{\cdot}2 \times \frac{{\cdot}00238}{2} \times 1{\cdot}25 \times (170)^2 \times 2{\cdot}149.$$

Note that $2{\cdot}149 = \left(\frac{88}{60}\right)^2$ which is the factor to convert (m.p.h.)^2 to (ft. per second)^2.

Set L.H. 1 of scale C to 1·2 on scale D, move cursor to 119 scale C, bring L.H. 1 of scale C to cursor move cursor to 289 (which is 17^2) on scale C, bring R.H. 1 of scale C to cursor, move cursor to 2·149 on scale C and under cursor read on scale D 11,090 lbs.

$$\therefore\ \text{Wing Lift} = L = 11090 \text{ lbs.}$$

To find angle of bank (ø).

Now, $\text{Cos } ø = \frac{W}{L}$ where W = weight of plane and L = lift $= \frac{2550}{11090}$

Set 255 on right hand part of scale B under 1109 on the *extended* scale to the right of the R.H.1 of scale A, and under R.H. 1 of scale A read on scale B :—

$$0{\cdot}23 = \text{Cos } ø$$

Now refer to the angle on the S scale indicated by the Index on the right hand window on the back of the scale. This is 13° 20′ and is the angle whose *sine* is ·23.

The angle whose *Cosine* is ·23 is obviously the complement of this and is of course $90° - 13° 20' = 76° 40' = \phi =$ angle of bank.

Over the L.H. 1 of scale B project cursor to scale A and read

$$4 \cdot 35 = \text{load factor during turn} = \frac{L}{W} = \frac{11090}{2550}$$

This is an interesting example of where 3 separate answers may be read off from a single setting of the slide rule.

(9) FINDING THE REVOLUTIONS PER MINUTE OF THE ENGINE OF A STATIONARY AEROPLANE.

An aeroplane is fitted with an engine whose normal horse power is 300 at 2,000 revs. per minute. Assuming that the Engine Torque at full throttle is constant, find the engine r.p.m. at full throttle when the aeroplane is stationary on the ground. The Air Screw Diameter is 10 ft. and the value of the Torque Coefficient (C_Q) when $V/_mD = 0$ is estimated at 0·22.

Given b.h.p. = 300, n = number of revs. per sec. = 33·33 ; D = Diameter of Air Screw = 10 ft. ; V = speed of flight ; Q = Engine Torque in ft. lbs. ; C_Q = Torque Coefficient ; and P = brake horse power of engine.

$$Q = \frac{550\,P}{2\pi n} = \frac{550 \times 300}{2 \times \pi \times 33 \cdot 33}$$

Set cursor to 55 on P scale and under cursor read 1·75 on D scale, bring 2 on C scale to cursor and move cursor to 3 on C scale, now bring 33·33 on C scale to cursor and under R.H. 1 of C scale read on D scale, 789 ft. lbs.

∴ Torque of engine = Q = 789 ft. lbs. and is constant. Now when V = 0 then $Q = C_Q \tfrac{1}{2} \rho n^2 D^5$ (ρ = Fluid density of air as before).

$$\therefore\ n^2 = \frac{Q}{C_Q \tfrac{1}{2} \rho D^5}$$

$$\text{whence } n = \sqrt{\frac{789}{\cdot 002 \times \cdot 00119 \times 10^5}}$$

Set L.H. 1 of scale C to 22 on scale D, move cursor to 119 on scale C, bring 789 on scale C to cursor and over L.H. 1 of scale D, read 300 on scale C. Now set cursor to 300 on left hand part of scale A and under cursor on scale D read 17·35 r.p.s.

Now bring R.H. 1 of scale C to cursor, move cursor to 60 on scale C and under cursor on scale D read 1044.

∴ n = 1044 revs. per minute.

(10) FINDING THE PARASITE DRAG OF AN AEROPLANE.

Find the parasite drag of an aeroplane flying at 140 m.p.h. for which the following data are supplied :—

Drag of fusilage 1/10th scale at 50 f.p.s. .... = 0·28 lb.
Drag of undercarriage $\tfrac{1}{4}$ scale at 50 f.p.s. .... = 0·42 lb.
Struts, wires, tail unit, etc. estimated full scale at
150 f.p.s. .... = 50 lbs.
D_1 = Drag of model (in lbs.), D_2 = full scale drag (in lbs.)

V = speed of flight (in ft. per sec.), Scale 2 = Full scale.

Scale 1 = Scale of model. Here again $2{\cdot}149 = \left(\frac{88}{60}\right)^2$ is used in the conversion factor from (m.p.h.)^2 to (ft. per sec.)^2.

V_1 = speed of model, V_2 = speed of full scale plane.

$$\text{Now } D_2 = \frac{D_1 \times \left(\frac{\text{Scale 2}}{\text{Scale 1}}\right)^2 \times (V_2)^2}{(V_1)^2}$$

$$\therefore \text{ Fusilage drag} = \frac{0{\cdot}28 \times 10^2 \times 140^2 \times 2{\cdot}149}{50^2}$$

Set cursor to 14 on scale D and under cursor on scale A read 196 ($14^2 = 196$). Note this down.

Now set L.H. 1 of scale C to 28 on scale D move cursor to 196 on scale C bring 25 (*i.e.* 5^2) on scale C to cursor, now move cursor to 2·149 on scale C and under cursor on scale D read.

473 lbs. = Fusilage Drag.

Using same Formulae,

$$\text{Undercarriage Drag} = \frac{0{\cdot}42 \times 4^2 \times 140^2 \times 2{\cdot}149}{50^2}$$

Set L.H. 1 of scale C to 42 on scale D move cursor to 16 (*i.e.* 4^2) on scale C bring 25 (*i.e.* 5^2) on scale C to cursor, set cursor to 196 (*i.e.* 14^2) on scale C, bring R.H. 1 of scale C to cursor, now move cursor to 2·149 on scale C and under cursor on scale D read 113 lbs.

$\therefore$ Undercarriage Drag = 113 lbs.

Still using same formula

$$\text{Remainder drag} = \frac{50 \times 140^2 \times 2{\cdot}149}{150^2}$$

Set L.H. 1 of scale C over 5 on scale D, bring cursor to 196 on scale C, bring 225 (= 15^2) on scale C to cursor, move cursor to 2·149 on scale C and under cursor on scale D read 93·5 lbs.

$\therefore$ remainder drag = 93·5 lbs.

$\therefore$ Total drag = 473 lbs. + 113 lbs. + 93·5 lbs. = 679·5 lbs.

THE "TECHNICAL" SERIES OF SLIDE RULES

THE STANDARD RULE

The Rule for General Use

THE SURVEYOR'S RULE

The Rule for Trigonometrical Calculations, and Aerial and Sea Navigation

THE AERONAUTICAL RULE

The Practical Rule for the Aeronautical and Electrical Engineer

THE POCKET PRECISION RULE

The rule equipped with 5″ scales but giving a 10″ scale accuracy with magnified cursor

THE TWENTY RULE

A 10″ rule giving a 20″ scale accuracy with magnified cursor

THE STANDARD POCKET RULE

The handy rule for the pocket with 5″ scales

Printed by
SENTINEL PRINTING
COMPANY, LTD.
LONDON, W.1